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XIV. Lunar and Planetary Instruments

SPACE SCIENCES DIVISION

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A. Mass Spectrometer Gaussmeter, *J. R. Locke*

1. Introduction

In support of mass spectrometer instrumentation, a gaussmeter has been designed and is presently being tested. This gaussmeter will be used to monitor the field of a 5-kilogauss permanent magnet essential to mass identification. To detect masses in the 10 to 120 range, it is necessary to know the permanent magnet's field strength to better than 0.5%.

A Hall-effect probe is used to monitor the field. The probe is driven by a transformer-coupled, square-wave-modulated, field-strength-dependent current source. The output of the Hall-effect probe is ac-coupled into a voltage gain stage, synchronously demodulated, filtered, dc-amplified, and fed back to control the current source.

2. Accuracy Consideration

To understand the significance of the magnetic field in connection with the mass spectrometer, consider Fig. 1, which is a simplified diagram demonstrating the principle that is involved. The potential energy of the ionized particle of mass m and charge q is converted into kinetic energy by

$$\frac{1}{2} m v_f^2 = qV \tag{1}$$

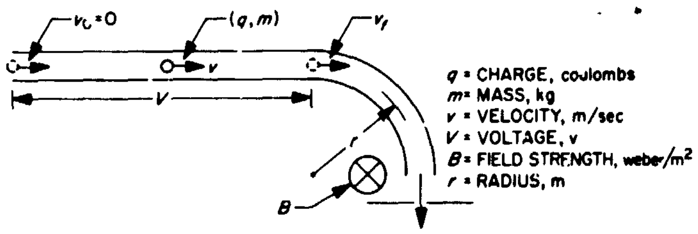


Fig. 1. Mass spectrometer principle

The B-field causes the charged particle to pass through a radius r:

$$r = \frac{m v_f}{qB} \tag{2}$$

$$\frac{m}{q} = \frac{2V}{v_f^2} \tag{1a}$$

$$v_f^2 = \left[\frac{rB}{\left(\frac{m}{q}\right)}\right]^2 \tag{2a}$$

Substituting Eq. (2a) into Eq. (1a),

$$\frac{m}{q} = \frac{r^2 B^2}{2V}$$

Considering only monovalent substances, $q = c$, where e is the charge of one electron, and

$$m = \frac{er^2B^2}{2V}$$

Since only the mass and B-field relationship is considered, let

$$\frac{er^2}{2V} = \phi$$

Then

$$m = \phi B^2$$

$$\frac{\partial m}{\partial B} = 2\phi B$$

$$\frac{\partial m}{m} = \frac{2\partial B}{B}$$

Therefore, with all other factors constant, to resolve 1 part in 120, it is necessary to know the B-field 1 part in 240, or to 0.42%

3. Instrumentation

Fig. 2 is a block diagram of the gaussmeter and Fig. 3 is the schematic identifying the components that will be referred to in the following analyses.

a. The Hall-effect probe. If an electric field is imposed upon a conductor or semiconductor, the electrons (or holes) drift from one current electrode to the other. If a magnetic field B is imposed in the transverse direction, the electrons are deflected by the Lorentz force acting upon them and a charge builds up on the boundary of the current-carrying element until the repulsive force, due to the charge, balances the Lorentz force. The resultant electrostatic field across the width of the conductor (or semiconductor) gives rise to a voltage which can be expressed as:

$$V_H = \frac{R_H}{t} (I_c B \sin \Theta)\, 10^{-8}\ \text{v}$$

where

R_H = Hall coefficient, cm^3/coulomb

t = element thickness, cm

I_c = control current, amp

B = magnetic field density, gauss

Θ = angle between I_c and B

If Θ = 90 deg, then $V_H = KI_cB$ where K = v/amp-kilogauss.

b. Design considerations. The Hall-effect probe chosen for this application is the Bell BH-200. This device has a sensitivity constant of 0.08 v/amp-kilogauss and is operated at a nominal control current of 100 ma. In a 5-kilogauss field, the resultant voltage is 40 mv. To measure this level within 0.42% (170 μv) precludes making a straight dc voltage measurement because of amplifier dc offset current and offset voltage variation with temperature. The alternatives are to use a dc control current

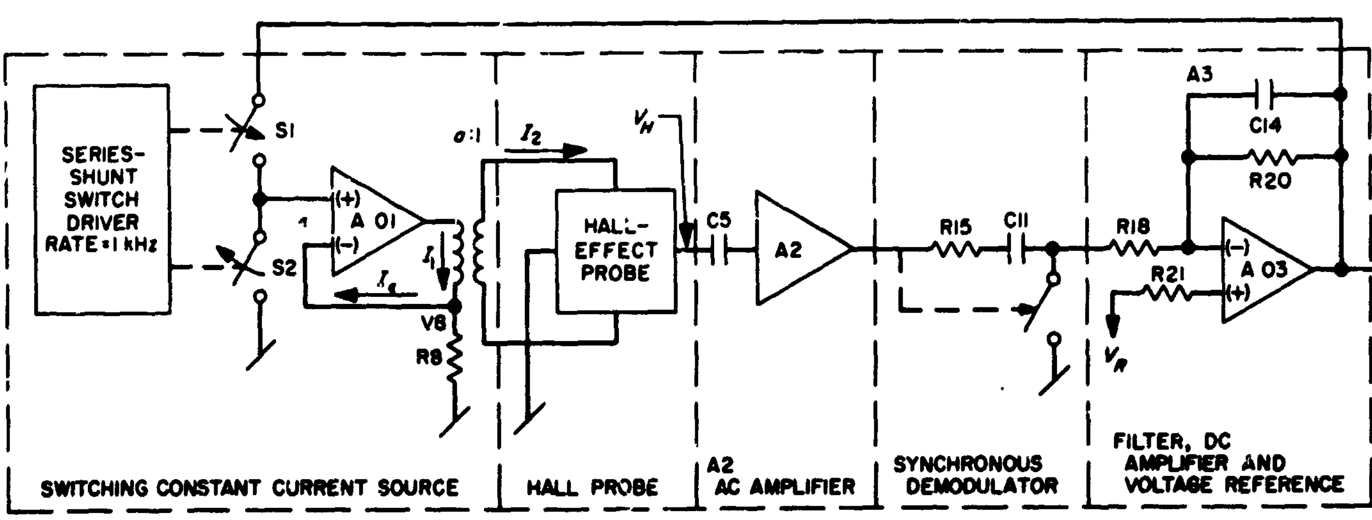
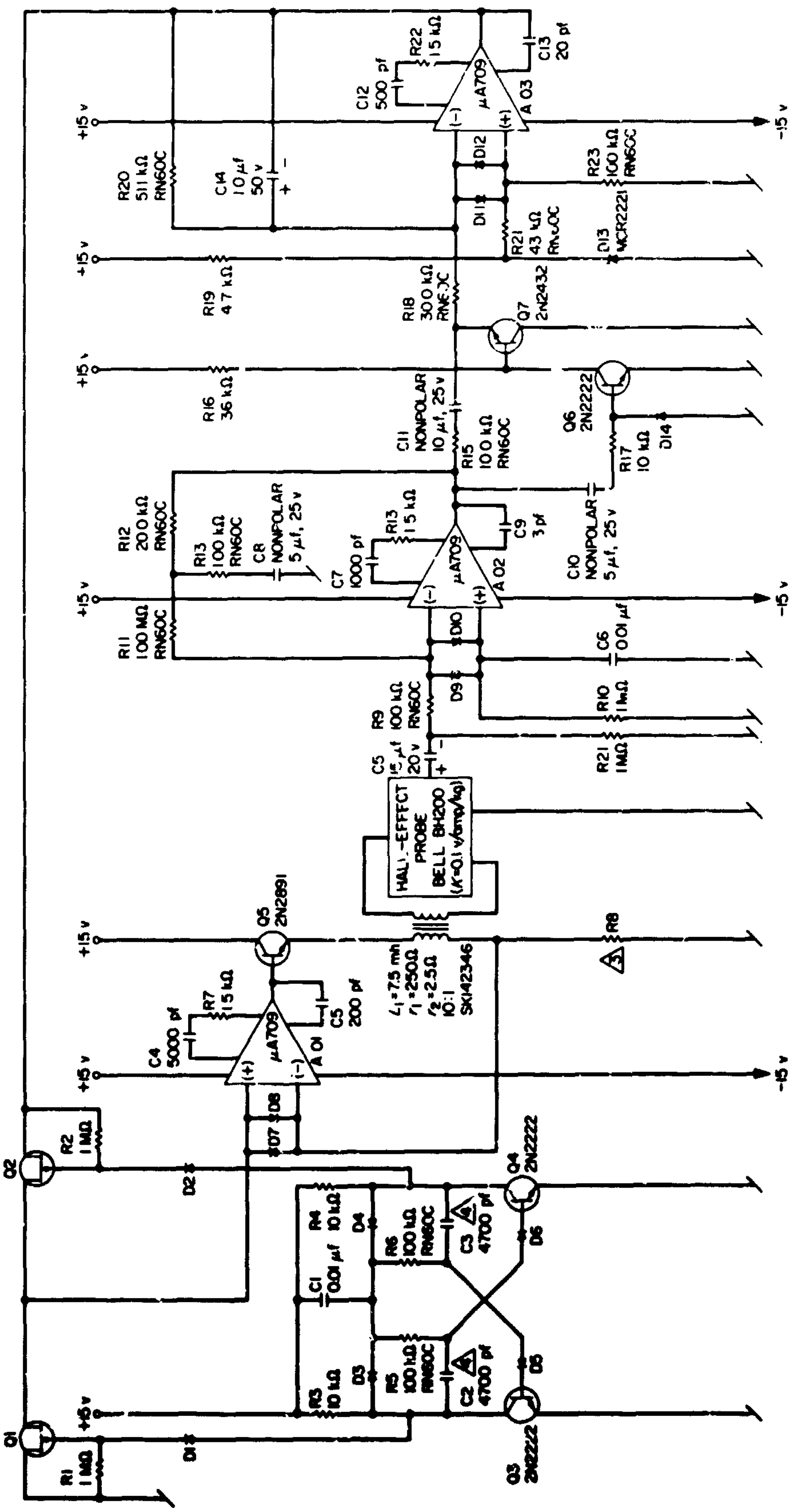


Fig. 2. Block diagram of gaussmeter



NOTES:
1. ALL DIODES IN914 UNLESS OTHERWISE NOTED
2. ALL RESISTORS ARE 5%, 1/4 w UNLESS OTHERWISE SPECIFIED
3. CHOOSE R8 TO TRACK THE TEMPERATURE COEFFICIENT OF THE HALL-EFFECT PROBE
4. C2 AND C3 CHOSEN TO TRACK EACH OTHER WITHIN 10 ppm/°C

Fig. 3. Mass spectrometer gaussmeter

and a chopper stabilized amplifier, or an ac control current and an ac amplifier with demodulator. The latter approach was decided upon for the following reasons:

(1) The lowest mass spectrometer system supply voltages are ± 15 v, and it is therefore desirable to transformer-couple the control current into the probe to minimize power consumption.

(2) Transformer coupling provides required isolation between the control current terminals and the voltage output terminals.

4. Design Theory

a. The transfer function. This section is devoted to developing the transfer function that relates the gaussmeter output voltage to the magnetic field. Then, using this transfer function, an analysis will be made of the error contributions.

Consider the Hall probe voltage expression:

$$V_H = KI_c B \tag{3}$$

Referring to Fig. 2,

$$I_2 = I_c = aI_1 = a\left(I_\epsilon + \frac{V_o - \epsilon}{R_s}\right) \approx \frac{aV_o}{R_s} \tag{4}$$

since $I_\epsilon \approx 0.5\,\mu$a (compared with 10 ma) and $\epsilon \approx 1.0$ mv (compared with 7.0 v) for switch 1 closed and switch 2 open. For switch 1 open and switch 2 closed, $I_c = 0$.

Because the Hall probe is ac-coupled into amplifier A2, the peak-to-peak input voltage is

$$V_{P\text{-}P} = V_H = \frac{KaV_o B}{R_s} \tag{5}$$

This voltage is amplified by ac amplifier A2 and becomes the demodulator input voltage

$$(V_{in})_{DEM} = \frac{KaV_o BA_2}{R_s} \tag{6}$$

To simplify the analysis of the demodulator, the following assumptions are made:

(1) Chopper transistor Q7 switches synchronously with the input signal $(V_{in})_{DEM}$.

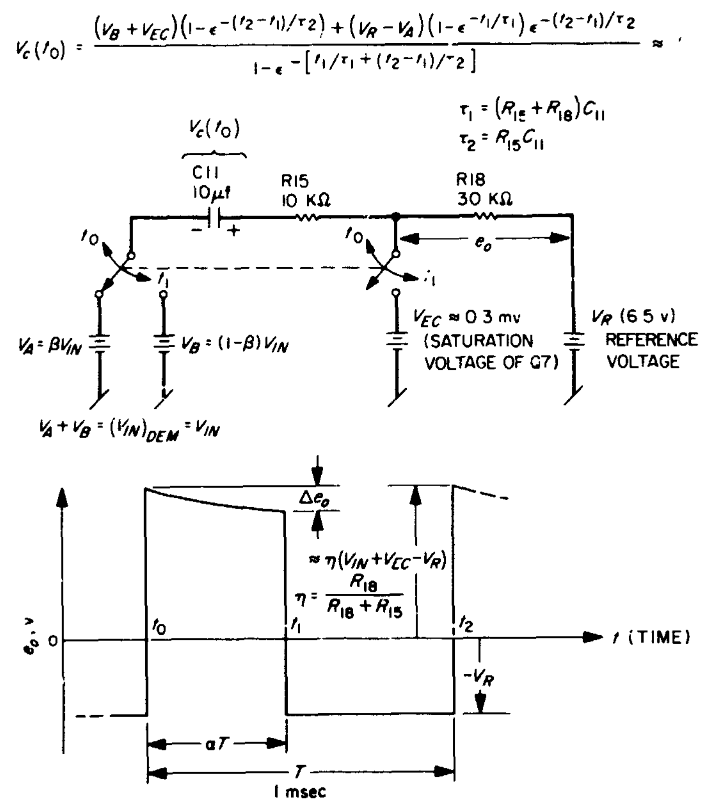


Fig. 4. Demodulator circuit model and output waveform

(2) The input voltage $(V_{in})_{DEM}$ is idealized as a perfect square wave (i.e.; no droop, $t_{FALL} = t_{RISE} = 0$).

(3) The summing point of amplifier A3 is at the same voltage as V_R.

(4) The output impedance of amplifier A2 and the saturation resistance of the chopper transistor Q7 can be neglected.

Using the indicated assumptions, the demodulator may be represented by the model shown in Fig. 4. Fig. 4 also indicates the output voltage that will be developed across the input resistor (R18) of amplifier A3.

To further simplify the analysis, the exponential decay of e_o will be linearized. Then

$$\Delta V_c = \frac{i_c}{C_{11}} \Delta T \tag{7}$$

where ΔV_c is the voltage change across C_{11} during the interval t_0 to t_1.

The point at which the demodulator analysis begins, defined as t_0, assumes that capacitor C11 has charged to its steady state value and that the voltage across it is approximately V_B. Then

$$i_c \approx \frac{V \;\; + V \;\; + V_{EC} - V_R}{R_{15} + R_{18}}$$

$$\Delta T = \alpha T$$

$$\Delta V_c \approx \frac{V_{in} + V_{EC} - V_R}{(R_{15} + R_{18})\, C_{11}}\, \alpha T$$

$$\Delta V_c \approx (V_{in} + V_{EC} - V_R)\, \alpha \frac{T}{\tau_1} \tag{8}$$

where

$$\tau_1 = (R_{15} + R_{18})\, C_{11}$$

To establish a relationship that relates the peak-to-peak input voltage of the demodulator to the dc (average) voltage across resistor R18, the following relationship is stated:

$$\psi \equiv \frac{(e_o)_{AVG}}{V_{in}} = \frac{1}{V_{in}T}\int_{t_0}^{t_1} e_o dt + \frac{1}{V_{in}T}\int_{t_1}^{t_2} e_o dt \tag{9}$$

$$\psi \approx \eta\alpha\left(1 + \alpha\frac{T}{2\tau_1}\right) + \frac{1}{V_{in}}\left\{\eta V_{EC}\, \alpha\left(1 - \alpha\frac{T}{2\tau_1}\right) + V_R\left[\eta\, \alpha^2 \frac{T}{2\tau_1} + \alpha(1 - \eta) - 1\right]\right\} \tag{10}$$

For convenience of handling, ψ will be expressed as

$$\psi = D + \frac{H}{V_{in}} \tag{11}$$

It is desirable that Eq. (10) be simplified. To do this, the following parameter values will be utilized:

$$\eta = \frac{R_{18}}{R_{18} + R_{15}} = 0.75$$

$$\alpha = 0.5$$

$$T = 1 \text{ msec}$$

$$\tau_1 = 400 \text{ msec}$$

$$V_{EC} = 1 \text{ mv (max)}$$

$$V_R = 4.6 \text{ v}$$

Substituting these values into Eq. (10) results in the following:

$$\psi = (0.75)(0.5)\left[1 + \frac{(0.5)(1 \times 10^{-3} \text{ sec})}{2(400 \times 10^{-3} \text{ sec})}\right] + \frac{1}{V_{in}}\left\{(0.75)(1 \times 10^{-3} \text{ v})(0.5)\left[1 - \frac{0.5(1 \times 10^{-3} \text{ sec})}{2(400 \times 10^{-3} \text{ sec})}\right] + 4.6 \text{ v}\left[\frac{(0.75)(0.25)(1 \times 10^{-3} \text{ sec})}{2(400 \times 10^{-3} \text{ sec})} + 0.5 \times 0.25 - 1\right]\right\} \tag{12}$$

By inspecting corresponding terms, it can be seen that Eq. (10) can be written as follows with very little loss in accuracy:

$$\psi = \eta\alpha + \frac{V_R}{V_{in}}[\alpha(1 - \eta) - 1] = \frac{(e_o)_{AVG}}{V_{in}} \tag{10a}$$

Having established a relationship between the output of amplifier A2 and the input of amplifier A3, the system output voltage may be written using Eq. (6) as the input to the demodulator. Thus,

$$(V_{in})_{DEM} = \frac{KaV_o BA_2}{R_8} \tag{6}$$

$$(e_o)_{AVG} = (V_{in})_{DEM}\, \psi = V_{in}\, D + H \tag{11a}$$

where the output of amplifier A3 is given as

$$V_o = V_R + A_3 (e_o)_{AVG} \tag{13}$$

Combining Eqs. (6), (11a), and (13) yields

$$V_o = V_R + A_3 \frac{KaV_o BA_2 D}{R_8} + A_3 H$$

Collecting terms, solving for V_0, and substituting the values for D and H determined in Eq. (10a) gives the transfer function relating the gaussmeter output to the B-field:

$$V_o = \frac{V_R\{1 - A_3[1 - \alpha(1 - \eta)]\}}{1 - \frac{A_3 A_2 KaB\eta\alpha}{R_8}} \tag{14}$$

b. Thermal analysis. For purposes of future reference the following nominal values will be used:

$$V_R = 4.6 \text{ v}$$
$$A_1 = -17$$
$$A_2 = 210$$
$$V_D = 0 \text{ v}$$
$$K = 0.08 \text{ v/amp-kilogauss}$$
$$a = 10$$
$$B = 5 \text{ kilogauss}$$
$$\eta = 0.75$$
$$\alpha = 0.50$$
$$R_s = 700\,\Omega$$
$$V_o = 7.2 \text{ v}$$

To simplify the following analyses, Eq. (14) can be approximated as

$$V_o \approx \frac{\lambda \{V_R [1 - \alpha(1-\eta)] + V_D\}}{A_2 a B \eta \alpha} \tag{14a}$$

where

$$\lambda \equiv \frac{R_R}{K}$$

without introducing any substantial error. V_D has been added to represent the voltage and current offset error associated with amplifier A3.

Since all of the parameters in Eq. (14a) may be considered temperature sensitive, with the exception of a and η which will be neglected, we may then write

$$\Delta V_o \approx \left(\frac{dV_o}{dT}\right)\Delta T$$
$$= \left(\frac{\partial V_o}{\partial V_R}\frac{dV_R}{dT} + \frac{\partial V_o}{\partial \alpha}\frac{d\alpha}{dT} + \frac{\partial V_o}{\partial \lambda}\frac{d\lambda}{dT} + \frac{\partial V_o}{\partial A_2}\frac{dA_2}{dT} + \frac{\partial V_o}{\partial V_D}\frac{dV_D}{dT} + \frac{\partial V_o}{\partial B}\frac{dB}{dT}\right)\Delta T \tag{15}$$

The mass spectrometer is required to operate from 0 to +50°C with a maximum allowable uncertainty in the B-field of 0.42%; therefore

$$\frac{\Delta V_o}{V_o} \times 100 = 0.42\%$$
$$= \frac{100}{V_o}\left(\frac{\partial V_o}{\partial V_R}\frac{dV_R}{dT} + \frac{\partial V_o}{\partial \alpha}\frac{d\alpha}{dT} + \frac{\partial V_o}{\partial \lambda}\frac{d\lambda}{dT} + \frac{\partial V_o}{\partial A_2}\frac{dA_2}{dT} + \frac{\partial V_o}{\partial V_D}\frac{dV_D}{dT}\right)\Delta T \tag{16}$$

with $\quad \dfrac{\partial V_o}{\partial B}\dfrac{dB}{dT} \quad$ excluded.

c. A3 input offset error V_D. The amplifier model shown in Fig. 5 was used to analyze the offset voltage and current drift. The resultant output drift is given by

$$\Delta V_D = \left(\Delta I_{OS} \pm \frac{\Delta V_{OS}}{R_S}\right)(R_{18} + R'_{21}) \tag{17}$$

where

$$R_S \equiv (R_{18} + R'_{21}) \,||\, R_D \,||\, R_{20} \approx 45\text{ k}\Omega, \; R'_{21} = R_{21} \,||\, R_{23}$$

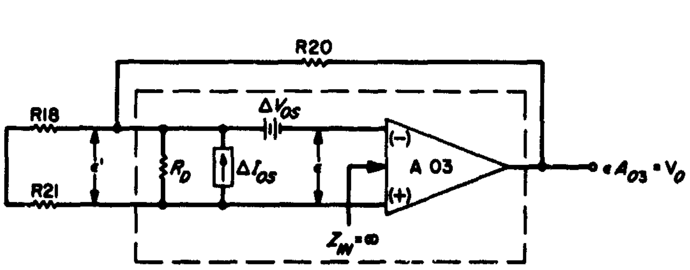
From the Fairchild μA 709 data sheet,

$$\Delta I_{OS} = 20 \text{ na, and } \Delta V_{OS} = 150\ \mu\text{v for } T = 0 \text{ to } +50°\text{C}.$$

Therefore,

$$\frac{dV_D}{dT}\Delta T \approx \Delta V_D = \pm\left(20 \text{ na} + \frac{150\mu\text{v}}{45\text{ k}\Omega}\right) 60\text{ k} = \pm 1.40 \text{ mv}$$

$$\delta V_D \approx \frac{\partial V_o}{\partial V_D}\Delta V_D \tag{18}$$



ΔI_{OS} AND ΔV_{OS} ARE THE CHANGES IN DIFFERENTIAL OFFSET CURRENT AND VOLTAGE THAT RESULT FOR A CHANGE IN TEMPERATURE ΔT (POLARITIES SHOWN ARE ARBITRARY)

Fig. 5. Amplifier drift model

where

$$\frac{\partial V_o}{\partial V_D} \approx \frac{-V_o}{V_R\,[1 - \alpha\,(1 - \eta)] + V_D}$$

Therefore,

$$\delta V_D \approx \frac{(7.2\text{ v})\,(\pm 1.40\text{ mv})}{(4.6\text{ v})\,(0.875)} = \pm 2.50\text{ mv}$$

d. Reference voltage error V_R. The reference voltage is fixed by dividing down the Zener diode (D13) voltage V_Z by resistors R21 and R23. Then

$$V_R = kV_Z$$

where

$$k \equiv \frac{R_{21}}{R_{23} + R_{21}}$$

If R23 has a maximum temperature coefficient (TC) of +50 ppm/°C and R21 has the smaller TC and tracks R23 to within 30 ppm/°C, the error caused at the output (ΔV_o) is 14 nv and may therefore be neglected. The Zener diode is then the primary error determiner. The Zener diode chosen (MCR2221) has a maximum TC of $|\pm 5\text{ ppm/°C}|$.

For $\Delta T = 50°\text{C}$,

$$\Delta V_R = k\Delta V_Z = (0.7)\,(6.5\text{ v})\,(\pm 5 \times 10^{-6}/°\text{C})\,(50°\text{C}) = \pm 1.14\text{mv}$$

$$\frac{\partial V_o}{\partial V_R} = \frac{V_o}{V_R}$$

$$\delta V_R = \frac{V_o}{V_R}\,\Delta V_R = \frac{7.2\text{ v}}{4.6\text{ v}} \times \pm 1.14\text{ mv} = \pm 1.78\text{ mv} \tag{19}$$

e. Switching time symmetry α. The switching symmetry is determined by how well matched and how closely the timing components of the multivibrator track their counterparts. Eq. (20) is an expression derived to calculate the half-period of oscillation of the multivibrator:

$$\frac{T}{2} = R_5C_2 \ln\left[\frac{2E_{BB}K_1 - 2K_1V_{D4} + (K_2 - 2K_1K_2 - 1)\,V_{D5} + V_{BE3}\,(K_2 - 2K_1K_2 - 1)}{E_{BD}K_1 + (1 - K_1)\,V_{D4} + (K_2 - K_1K_2)\,V_{D5} + V_{BE3}\,(K_2 - K_1K_2) - V_{D6} - V_{BE4}}\right] \tag{20}$$

where

$$K_1 = \frac{R_5 \,||\, R_6}{R_5 \,||\, R_6 + R_4}$$

and

$$K_2 = \frac{R_5}{R_5 + R_6}$$

The K terms (K_1 and K_2) are relatively temperature insensitive and can be ignored. More significant, however, is the tracking of V_{D5} with V_{D6} and V_{BE3} with V_{BE4}, since they appear only in the denominator of the timing expression. By far the most significant is the matching of R5 with R6 and C2 with C3. Assuming most of the dissymmetry is caused by the tracking of C2 and C3, they are chosen so as to have not more than 0.05% differential change. Using this figure,

$$\frac{\partial V_o}{\partial \alpha} = -\frac{1}{[1 - \alpha\,(1 - \eta)]}\,\frac{V_o}{\alpha} \tag{21}$$

and

$$\delta\alpha \approx \frac{\partial V_o}{\partial \alpha}\,\Delta\alpha = -\frac{1}{[1 - \alpha\,(1 - \eta)]}\,\frac{\Delta\alpha}{\alpha}\,V_o$$

Then

$$\delta\alpha \approx \frac{-1}{1 - 0.5\,(1 - 0.75)} \times \mp 5 \times 10^{-4} \times 7.2\text{ v} = \pm 4.1\text{ mv}$$

f. Amplifier A2 error. Let the amplifier error of A2 be expressed as

$$\delta_A = \frac{\partial V_o}{\partial A_2}\,\frac{dA_2}{dT}\,\Delta T \tag{22}$$

$$\frac{\partial V_o}{\partial A_2} = \frac{-V_o}{A_2} \tag{23}$$

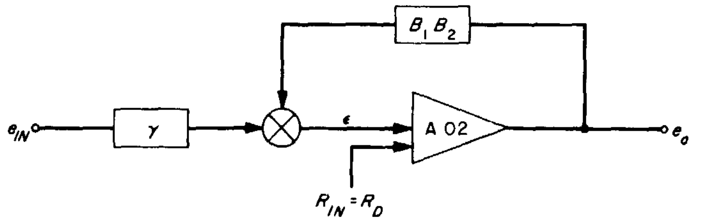


Fig. 6. Gain analysis model

Fig. 6 is the amplifier model used in deriving the closed-loop midband gain expression for A2. The summing point error voltage is given by

$$\epsilon = \frac{e_o}{A_{o2}} = e_{in}\,\gamma + B_1B_2e_o \qquad (24)$$

where

$$\gamma = \frac{R_D \,||\, R_{11}}{R_D \,||\, R_{11} + R_9} = \frac{R_9 \,||\, R_D \,||\, R_{11}}{R_9}$$

$$B_1 = \frac{R_D \,||\, R_9}{R_D \,||\, R_9 + R_{11}} = \frac{R_9 \,||\, R_D \,||\, R_{11}}{R_{11}}$$

$$B_2 = \frac{R_{13}}{R_{13} + R_{12}}$$

assuming that $R_{11} \gg R_{12} \,||\, R_{13}$.

Substituting the values of γ, B_1, and B_2 into Eq. (5) and solving for e_o/e_{in} (where $e_o/e_{in} \equiv A_2$) yields

$$A_2 = \frac{R_{11}}{R_9}\left(\frac{R_{13} + R_{12}}{R_{13}}\right)\left(\frac{F}{1-F}\right) = G\left(\frac{F}{1-F}\right) \qquad (25)$$

where

$$F = A_{o2}\frac{R_{13}}{R_{12} + R_{13}}\left(\frac{R_9 \,||\, R_D \,||\, R_{11}}{R_{11}}\right)$$

It will be assumed that the temperature coefficients and tracking of resistors R9, R11, R12, and R13 are such that their contribution to the gain change of A2 can be ignored and that the principal causes of change are due to A_{o2} and R_D.

$$\frac{dA_2}{dT} = \frac{G}{(1-F)^2}\frac{dF}{dT} \qquad (26)$$

$$\frac{\Delta A_2}{A_2} \approx \frac{1}{(1-F)F}\Delta F$$

$$\Delta A \approx A_2\frac{\Delta F}{(1-F)F} \approx \left(\frac{dA}{dT}\right)\Delta T$$

Therefore,

$$\delta A \approx \frac{-V_o}{A_2}A_2\frac{\Delta F}{(1-F)F} = \frac{-V_o}{(1-F)}\frac{\Delta F}{F} \qquad (27)$$

From temperature data taken on the Fairchild μA709, it is found that

$$A_{o2}\,(0°\mathrm{C}) = -50 \times 10^3 \text{ and } R_D\,(0°\mathrm{C}) = 300\ \mathrm{k\Omega}$$

$$A_{o2}\,(50°\mathrm{C}) = -40 \times 10^3 \text{ and } R_D\,(50°\mathrm{C}) = 550\ \mathrm{k\Omega}$$

$$F\,(0°\mathrm{C}) = -168,\ F(+50°\mathrm{C}) = -149,\ F(25°\mathrm{C}) \approx -158$$

Then

$$\delta A \approx -5.4\ \mathrm{mv}$$

g. Hall probe tracking error λ. The sensitivity constant K of the Hall probe has a reasonably large temperature coefficient (-800 ppm/°C). For this reason the other error contributors have been calculated first, so that the allowable probe tracking error would be fixed by the difference of the maximum allowable error and the sum of the other contributors. Thus,

$$\delta\lambda = V_o\left(\frac{0.42\%}{100}\right) - (\delta V_R + \delta V_D + \delta\alpha + \delta A_2)$$

$$= -30\ \mathrm{mv} + 13.8\ \mathrm{mv} \qquad (28)$$

Therefore,

$$\delta\lambda \approx V_o\frac{\Delta\lambda}{\lambda} = -16.2\ \mathrm{mv}$$

The actual tracking requirement is placed upon resistor R8. Assume that for a given increase in temperature the B-field of the magnet was to remain constant, but K decreased. Referring to the Hall voltage expression $V_H = KI_cB$, it can be seen that if the current were to increase in a manner that kept the KI_c product constant, no spurious signal would be introduced. This can be accomplished by decreasing R_8, since

$$KI_c = \frac{KV_o a}{R_8}$$

where V_o remains constant for V_H constant.

Therefore, assuming that the curve of R_s tracks the curve of K over the specified temperature range within the indicated tolerance, a 0.42% mass spectrometer gaussmeter seems attainable.